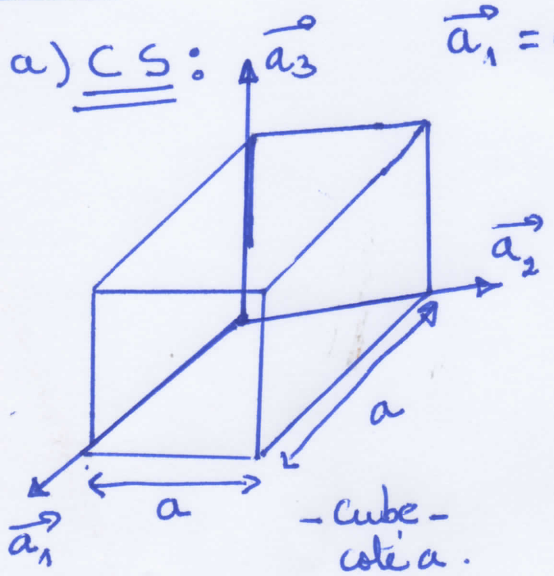
Série TD N° 01

*** Exo 1:**

1) a) CS:



$\vec{a_1} = a\vec{i}$; $\vec{a_2} = a\vec{j}$; $\vec{a_3} = a\vec{k}$.

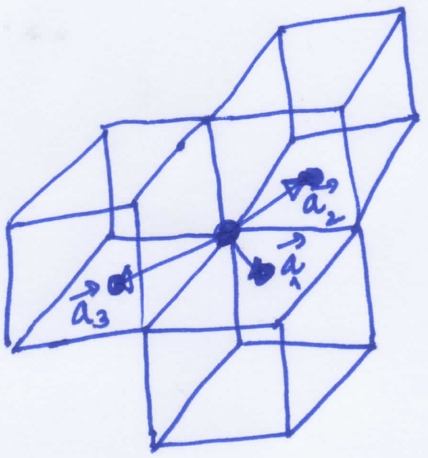
a : paramètre (côté).

b) CC:

$$\vec{a_1} = \frac{a}{2}(\vec{i} + \vec{j} - \vec{k}).$$

$$\vec{a_2} = \frac{a}{2}(-\vec{i} + \vec{j} + \vec{k})$$

$$\vec{a_3} = \frac{a}{2}(\vec{i} - \vec{j} + \vec{k}).$$

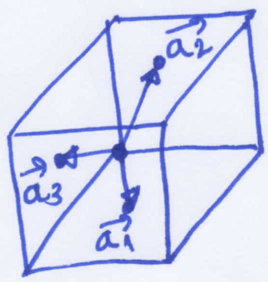


c) CFC:

$$\vec{a_1} = \frac{a}{2}(\vec{i} + \vec{j}).$$

$$\vec{a_2} = \frac{a}{2}(\vec{j} + \vec{k})$$

$$\vec{a_3} = \frac{a}{2}(\vec{i} + \vec{k}).$$



2) Calcul des volumes :

a) CS : $V_{CS} = |\vec{a}_1 \cdot (\vec{a}_2 \wedge \vec{a}_3)|$; $a_1 = a_2 = a_3 = a$.

$\vec{a}_2 \wedge \vec{a}_3 = a \cdot \vec{j} \cdot a \cdot \vec{k} = a^2 \vec{i} \Rightarrow V_{CS} = a\vec{i} \cdot a^2 \vec{i}$

$\Rightarrow \boxed{V_{CS} = a^3}$ $\quad \vec{i} \cdot \vec{i} = 1$.

b) CC : $V_{CC} = |\vec{a}_1 \cdot (\vec{a}_2 \wedge \vec{a}_3)|$

mais : $\vec{a}_2 \wedge \vec{a}_3 = \frac{a^2}{4} (-\vec{i} + \vec{j} + \vec{k}) \wedge (\vec{i} - \vec{j} + \vec{k})$

$$= \frac{a^2}{4} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \frac{a^2}{4} (2\vec{i} + 2\vec{j}).$$

$V_{CC} = \frac{a}{2} (\vec{i} + \vec{j} - \vec{k}) \cdot \frac{a^2}{4} (2\vec{i} + 2\vec{j})$

$= \frac{a^3}{8} \cdot 2 (\vec{i} + \vec{j} - \vec{k}) \cdot (\vec{i} + \vec{j})$.

$$\boxed{V_{CC} = \frac{a^3}{4}}$$

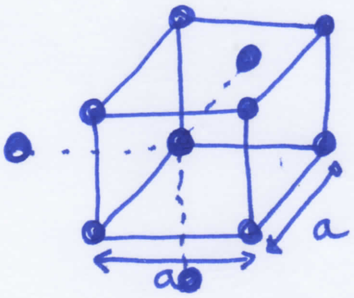
c) CFC : $\vec{a}_2 \wedge \vec{a}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \cdot \frac{a^2}{4} = \frac{a^2}{4} (\vec{i} + \vec{j} - \vec{k})$.

$V_{CFC} = \frac{a}{2} (\vec{i} + \vec{j}) \cdot \frac{a^2}{4} (\vec{i} + \vec{j} - \vec{k})$

$$\boxed{V_{CFC} = \frac{a^3}{4}}$$

3) Premiers proches voisins :

a) CS :



on considère un atome et on compte le nombre d'atomes proches à ce dernier

$$\boxed{n = 6}$$

b) CC :

$$\boxed{n = 8}$$

c) CFC :

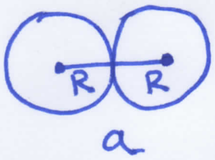
$$\boxed{n = 12}$$

4) Taux de remplissage : (compacité) :

$$\boxed{\tau = \frac{\text{Volume occupé}}{\text{Volume total}}}$$

Volume occupé → par les atomes
Volume total → de la maille

R : rayon



← atome

nombre de sphères / maille

$n = \frac{1}{8} \cdot 8 = 1$.

* CS : $V_{Total} = a^3$

~~$V_{occupé} = n \times \frac{4\pi}{3} R^3$~~

$V_{occupé} = n \cdot \frac{4\pi}{3} R^3$

or : $2R = a \Rightarrow R = \frac{a}{2}$.

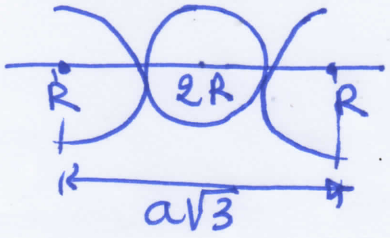
donc : $V_{occupé} = 1 \, \frac{4\pi}{3} \left(\frac{a}{2}\right)^3 = \frac{\pi}{6} a^3$

$$\tau = \frac{\frac{4}{3}\pi \left(\frac{a}{2}\right)^3}{a^3} \Rightarrow \boxed{\tau_{CS} = \frac{\pi}{6}}$$

* CC : $n = \frac{1}{8} \cdot 8 + 1 = 2$ atomes

$$\tau = \frac{2 \cdot \frac{4}{3}\pi R^3}{a^3}$$

mais :



$\sqrt{3}\,a = 4R \Rightarrow R = \frac{a\sqrt{3}}{4}$

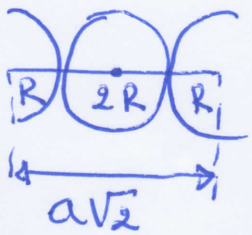
donc :

$$\tau_{cc} = \frac{\frac{8\pi}{3}\left(\frac{a}{4}\sqrt{3}\right)^3}{a^3}$$

$\boxed{\tau_{cc} = 0{,}68}$ ou bien 68 %.

* CFC :

$n = \frac{1}{2}\cdot 6 + \frac{1}{8}\cdot 8 = 4$



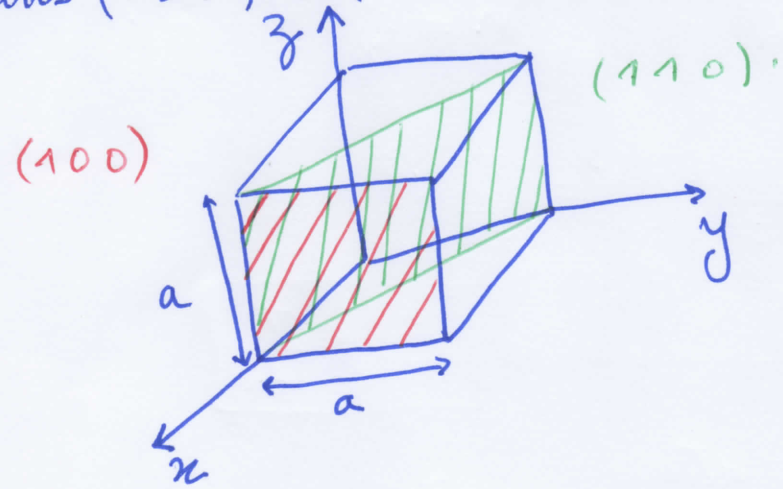
$4R = a\sqrt{2} \Rightarrow \boxed{R = \frac{a\sqrt{2}}{4}}$

$$\tau_{CFC} = \frac{4\cdot\frac{4}{3}\pi\left(\frac{a\sqrt{2}}{4}\right)^3}{a^3}$$

$\boxed{\tau_{CFC} = 0{,}74}$; 74 %

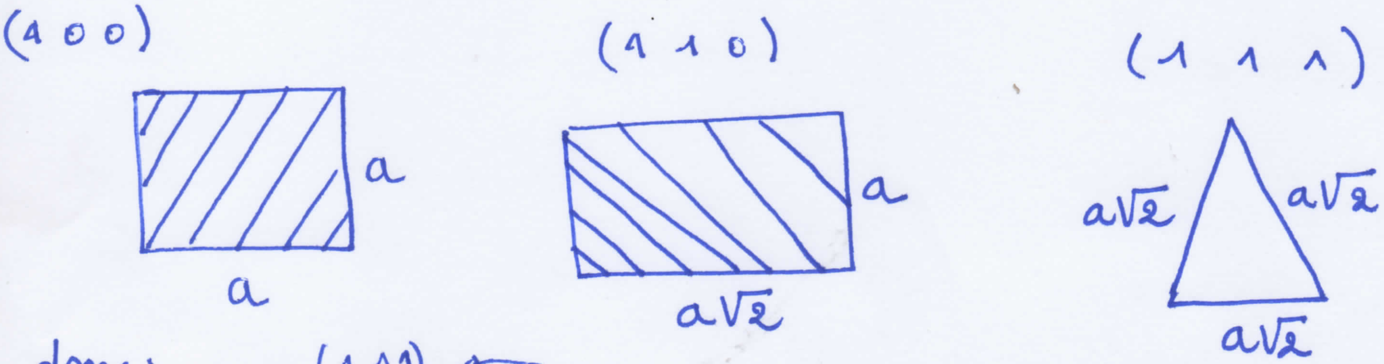
* Exo 2 :

1) Plans (1 0 0) et (1 1 0) pour un CC :

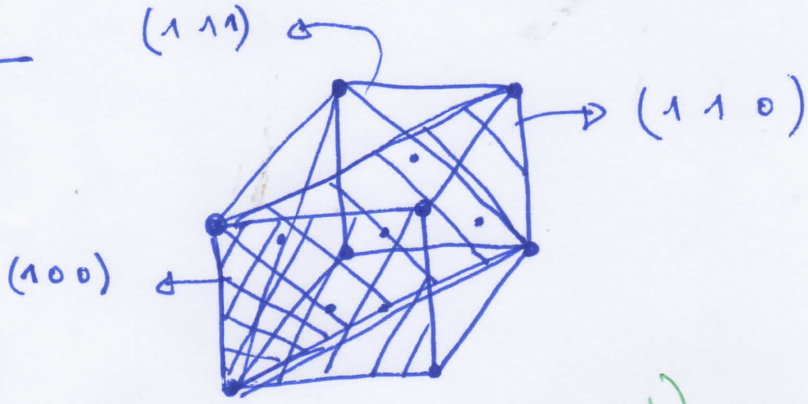


(voir la feuille).

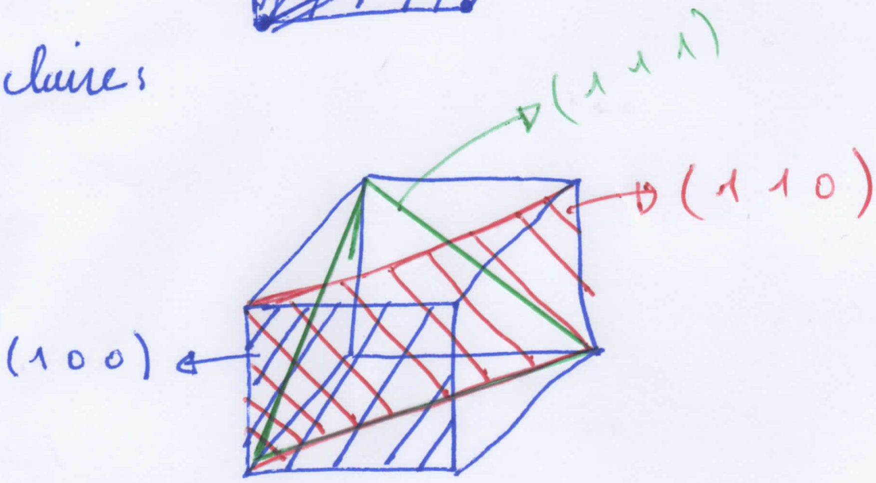
2) CFC : (100), (110) et (111).



donc :



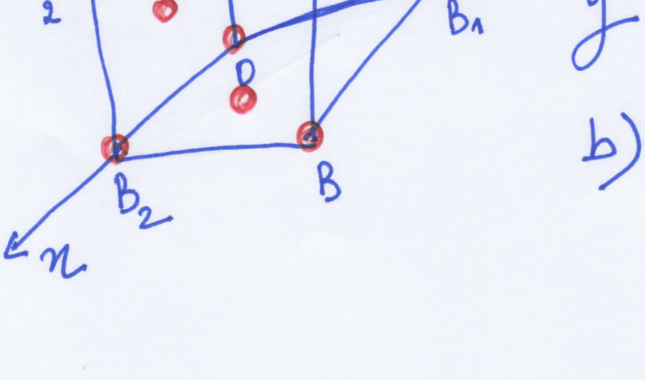
Plus claire :



*Exo 3 :

1)

a) CFC



{ 8 atomes aux sommets du cube.
{ 6 atomes aux centre des six faces du cube

b) Soit : 6 + 8 = 14

b) le cube contient :

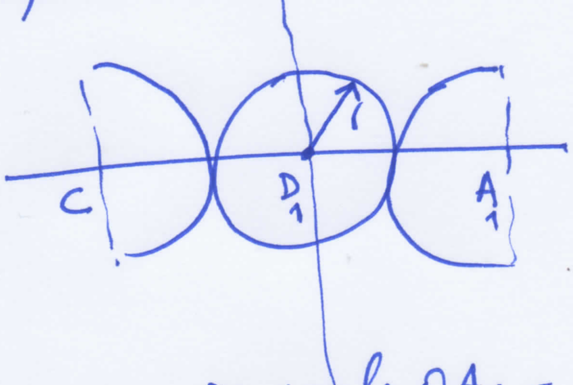
* 8 fois $\frac{1}{8}$ de sphères aux sommets du cube $\Rightarrow 8 \times \frac{1}{8} = 1$

Donc 1 sphère pleine.

* $6 \times \frac{1}{2} = 3$

au total : $1 + 3 = 4$ sphères pleines.

$$\left.\begin{array}{l} OA_1 = 4r \\ OA_1 = a\sqrt{2} \end{array}\right\} \Rightarrow \boxed{r = \frac{a}{2\sqrt{2}}}$$



Donc les atomes occupent le volume :

$$4\left(\frac{4}{3}\pi r^3\right) = \frac{\pi a^3}{3\sqrt{2}} = 0,74 a^3$$

$$\tau = \frac{V_{occ}}{V_T} = \frac{0,74 a^3}{a^3} \Rightarrow \boxed{\tau = 0,74}$$

2) le cuivre : $a = 3,61 \, A°$.

a) Dans le volume a^3, il y a 4 atomes

Dans $1 \, m^3 \longrightarrow N = \frac{4}{a^3}$.

Alors : $a = 3,61 \, A° = 3,61 \cdot 10^{-10} \, m$. $\Rightarrow \boxed{N = 8,5 \cdot 10^{28} / m^3}$

b) La masse volumique :

$A \longrightarrow N$

$$\frac{A}{N} = \frac{63,54 \cdot 10^{-3}}{6 \cdot 10^{23}} = 10,59 \cdot 10^{-26} \, Kg.$$

$10,59 \cdot 10^{-26} \times 8,5 \cdot 10^{28} \, Kg/m^3 \Rightarrow \boxed{9 \cdot 10^3 \, Kg/m^3} = 9 g/cm^3$.